Renegotiable Debt, Liquidity Injections, and Financial Instability*

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Abstract

This paper develops a debt-run model to study the effects of liquidity injections on debt markets in the presence of a renegotiation option. In the model, creditors decide when to withdraw their funding and equityholders can renegotiate the contract terms of debt. We show that when equityholders have a large bargaining power, liquidity injections into distressed firms can rather cause more aggressive runs from their creditors, hurting the debt value. This outcome occurs because equityholders can strategically utilize the renegotiation option as a bankruptcy threat, pushing down the debt value below the potential liquidation value of the firm. In such a scenario, a deterred default resulting from emergency capital injections could be detrimental to creditors.

Keywords: dynamic debt runs, renegotiation, liquidity injection, bailout

JEL Classifications: G01, G21, G33

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1 Introduction

The recent events such as the sudden collapse of Silicon Valley Bank and Signature Bank in March 2023 highlight the ongoing vulnerability of a financial system to debt runs which typically occur due to liquidity problems or coordination failures among creditors. Debt runs on non-traditional banks, such as investment banks, special purpose vehicles, and conduits, were also considered a major catalyst for the 2008 financial crisis as documented by Gorton and Metrick (2012), Covitz et al. (2013), and Schroth et al. (2014). Recognizing the devastating effects of debt runs not only on financial markets but also on the real economy, researchers have been seeking some effective measures to prevent a debt crisis caused by panic-driven runs. Nonetheless, whether a government's intervention in financial markets, such as liquidity injections or direct asset purchases, can effectively mitigate a debt crisis without causing any adverse effects is still questionable. For instance, various studies, including Eser and Schwaab (2016), Andrade et al. (2019), Acharya et al. (2019), Crosignani et al. (2020), Jang (2021), and Doh (2023) show mixed results regarding the impact of monetary and fiscal policies on financial stability.

In this paper, building on Leland and Toft (1996) and He and Xiong (2012a), we develop a dynamic debt-run model with a renegotiation option. Using this model, we show that liquidity injections into a financially constrained firm can rather trigger more aggressive runs from its creditors when equityholders can strategically renegotiate the contract terms of debt. The underlying reason is that when the value of debt is pushed below the potential liquidation value of a firm due to the renegotiation option of equityholders, allowing the firm to default earlier due to a liquidity problem can be actually beneficial to the firm's creditors. This result challenges the long-held view of Diamond and Dybvig (1983), who argue that emergency capital injections such as the provision of demand deposit insurance can effectively prevent bank runs. In the literature, Rochet and Vives (2004) and Liu (2016) further support this assertion by extending the model of Diamond and Dybvig (1983) in various directions.

In our model, a firm issues runnable debt to a large number of ex-ante identical creditors, which means each of those creditors can potentially request an early redemption at any date as long as the firm is alive. But, to create strategic uncertainty in a dynamic setup, we assume that each creditor observes the firm's fundamental only occasionally and makes the withdrawal decision only when she is awakened. This assumption is commonly adopted in the literature that studies dynamic coordination games; see, for instance, Frankel and Pauzner (2000) and He and Xiong (2012a). Also, here, we can interpret the date when a creditor makes the withdrawal decision as the maturity date of runnable debt, as in He and Xiong (2012a). The firm may fail to repay the principal amount to those running creditors, in which case, the firm is forced to default. As in He and Xiong (2012a), each creditor makes the withdrawal decision by considering not only the firm's current fundamental but also the withdrawal strategies of other creditors.

One important feature of the model is that the firm's equityholders can renegotiate the contract terms of debt at any time. Specifically, following Fan and Sundaresan (2000) and Davydenko and Strebulaev (2007), we assume that equityholders can renegotiate the contract terms with creditors through a debt-equity swap. For robustness, we also consider another renegotiation scheme known as strategic debt service examined in Mella-Barral and Perraudin (1997) and show the main results of this paper still hold. As commonly shown in the above papers, when equityholders have the renegotiation option, the value of debt generally cannot exceed the potential liquidation value of the firm, especially when equityholders have a large bargaining power. This outcome occurs because when the value of the outside option of creditors is equal to the liquidation value of the firm, equityholders can utilize the renegotiation option as a bankruptcy threat to compel creditors to accept the revised terms that eventually push down the debt value below the potential liquidation value of the firm.

When the value of debt lies below the potential liquidation value of the firm, creditors would be better off if the firm somehow defaults earlier due to the failure to meet the redemption requests of other creditors. As such, in this situation, if the government injects more liquidity into distressed firms to defer the liquidity-driven default, the debt value will be rather lowered, triggering more aggressive runs from creditors. In this regard, our paper underscores the need for a more comprehensive examination of the effectiveness of liquidity injection policies, such as the Term Asset-Backed Securities Loan Facility, Commercial Paper Funding Facility, and Secondary Market Corporate Credit Facility established by the Federal

Reserve during the 2008 financial crisis and the 2020 COVID-19 crisis, particularly through the renegotiation channel between borrowers and creditors.

This adverse outcome usually does not occur when equityholders do not have the renegotiation option as in He and Xiong (2012a). In particular, He and Xiong (2012a) show that if the government or a firm's parent company provides more reliable liquidity guarantees, creditors decide to run less aggressively, especially when the volatility of the firm's fundamental is at a normal level. When the volatility is severely high, He and Xiong (2012a) show that the liquidity backstop program may also cause more frenzy runs. But the underlying mechanism is different from the mechanism considered in this paper. In their paper, this negative outcome occurs because when the volatility is high, the debt value will be more flattened and thus can be also lower than the potential liquidation value of the firm. In our paper, such an outcome occurs because the debt value is squeezed down because equityholders can utilize their renegotiation option as a bankruptcy threat.

Our paper contributes to the literature on both debt runs and debt renegotiation. In the debt-run literature, Diamond and Dybvig (1983) first show that while demand deposit contracts can potentially solve the liquidity mismatch problem, those contracts can also leave depositors susceptible to self-fulling bank runs. Goldstein and Pauzner (2005) and Rochet and Vives (2004) incorporate this model into a global-game framework and pin down a unique equilibrium with bank runs. He and Xiong (2012a), Liu (2016), Wei and Yue (2020), and Liu (2023) further extend the bank-run model to study the dynamic interaction among creditors or the feedback effect between run decisions and liquidity. These papers generally show that liquidity backstop programs can mitigate debt runs and prevent inefficient bankruptcy events. However, none of these papers considers the intriguing interaction between the withdrawal decisions and the debt renegotiation. In this regard, further studies are warranted to investigate whether liquidity injections can indeed stabilize debt markets or can cause unexpected side effects more rigorously.

In the literature on debt renegotiation, researchers mainly focus on the effectiveness of debt renegotiation in alleviating the debt-overhang problem; see Mella-Barral and Perraudin (1997), Fan and Sundaresan (2000), Davydenko and Strebulaev (2007), and Wong and Yu (2021). Those papers commonly show that the renegotiation channel can alleviate the debt-overhang problem. Besides this issue, in our paper, we analyze whether the government's emergency funding can improve the stability of debt markets when equityholders have a renegotiation option.

The paper is organized as follows. In Section 2, we develop the main debt-run model with a renegotiation option. In Section 3, we solve the model. In Section 4, we discuss the model implications. In Section 5, we consider an alternative specification of the model. In Section 2, we conclude.

2 Model

This section develops a dynamic debt-run model in which equityholders have a renegotiation option, building on Leland and Toft (1996) and He and Xiong (2012a). Consider a firm with an asset in place that produces stochastic cash flows over time. Time is continuous. Considering a dynamic setup is natural because of the presence of the renegotiation option. The amount of cash flows produced at time t, denoted by x_t, evolves according to a geometric Brownian motion:

$$\frac{dx_t}{x_t} = \mu dt + \sigma dZ_t,$$

where μ is a growth rate, σ is a volatility, and Z_t is a standard Brownian motion. All agents are risk neutral and discount future cash flows at a constant risk-free rate r. The time-t cash flow, x_t, will be also called the time-t fundamental of the firm. In this setup, the first-best value of an unlevered firm is equal to

$$V^{FB}(x_t) = E_t \left[\int_t^{\infty} e^{-r(s-t)} x_s ds \right] = \frac{x_t}{r - \mu}.$$

We assume $r > \mu$ to ensure that the first-best value of the asset is finite.

The firm has issued a continuum of runnable debt with a total size normalized to 1. Each one unit of the debt contract pays a constant coupon c per unit of time. Each creditor can potentially request early redemption at any point in time. But we assume that each creditor is awakened according to a Poisson shock that arrives with an intensity λ. The creditor decides whether to request early redemption (that is, run or withdraw her funding) or not only when she is awakened. This assumption, which says that different creditors observe the firm's fundamental at different times, creates an intertemporal coordination problem among creditors as in He and Xiong (2012a). The assumption that creditors can request early redemption reflects the prevailing feature of debt contracts issued by non-bank financial institutions such as special purpose vehicles, investment banks, conduits, and bond mutual funds, as in He and Manela (2016). [1] Throughout, we focus on a symmetric threshold equilibrium, which means that, in equilibrium, each awakened creditor decides to run if and only if the firm's fundamental x_t is below some threshold, say, x_R, which is endogenously determined.

Here, if we alternatively assume that each creditor can make the withdrawal decision at every point in time, the withdrawal timing of all creditors will be synchronized. In such a case, multiple equilibria may arise as in global-game models with homogeneous information; see Carlsson and Van Damme (1993). Analyzing such a model is not the main interest of this paper.

If a creditor requests early redemption, the firm is required to pay back the promised amount of principal to that creditor. The principal amount is set to a constant F. We also impose the parameter condition that $\frac{c}{r} > F$ to ensure that when the firm's fundamental is sufficiently good so that there is no chance of default or renegotiation, early withdrawal is not optimal. Once the firm successfully pays back the principal amount, the firm issues another debt claim to keep the total size of debt constant.

When creditors request early redemption, the firm may fail to make the debt payment because of some liquidity problems. This liquidity-driven default is assumed to occur according to a Poisson shock that arrives with intensity θ, when and only when the firm faces withdrawing creditors, similar to He and Xiong (2012a). To justify this assumption, we first assume that when the firm is required to pay back only the coupons without facing

[1] Note that the main results of our paper can be also applied to situations where firms commit to rolling over all of their retiring debt claims and creditors do not have the withdrawal option, as in He and Xiong (2012b). But we adopt the setting of He and Xiong (2012a) to put more emphasis on the strategic interactions between equityholders and creditors. In that sense, we do not necessarily restrict our attention to only financial firms issuing runnable debt in this paper. Even when firms issue corporate bonds that do not include the withdrawal option, we can broadly interpret early withdrawals as premature sell-offs of bond investors managing bond funds, similarly as in Goldstein et al. (2017).

withdrawing creditors, even if the current cash flows are not sufficient to cover the coupon payment, the firm can always raise enough capital by issuing new equity to pay back those coupons, as long as the equity value of the firm remains positive. However, when the firm faces redemption requests from creditors, the firm may fail to repay the principal amounts to all those withdrawing creditors because we implicitly assume that a firm under runs encounters more severe liquidity problems. For instance, we may say that when a firm experiences runs, its manager may abscond some part of the firm's assets, leading investors to become reluctant to infuse additional capital into that company. We call the parameter θ the fragility of emergency funding. Or, we can interpret $\frac{1}{\theta}$ as the strength of emergency funding.

When the firm defaults due to the liquidity reason, the firm liquidates the asset immediately and receives

$$\frac{\alpha x_t}{r-\mu}$$

as the total bankruptcy proceeds, where α is the recovery rate of the asset in default. Then, creditors equally receive

$$L(x_t) = \min\left\{\frac{\alpha x_t}{r-\mu}, F\right\},$$

and equityholders equally receive the residual amount, which is equal to

$$H(x_t) = \max\left\{\frac{\alpha x_t}{r-\mu} - F, 0\right\}.$$

That is, when the liquidation value of the asset is lower than the promised amount of principal, the creditors receive all the bankruptcy proceeds; otherwise, the bankruptcy proceeds to creditors are truncated by the principal amount.

An important feature of this model is that equityholders can initiate renegotiation with creditors on the contract terms of debt. For simplicity, we first consider a debt-equity swap as the renegotiation scheme as in Fan and Sundaresan (2000) and Davydenko and Strebulaev (2007). In Section 5, we also consider another renegotiation scheme such as strategic debt service examined by Mella-Barral and Perraudin (1997) and Fan and Sundaresan (2000) as well. Specifically, at any point in time, the firm's equityholders can offer a debt-equity swap to creditors. Under this offer, if accepted, all debt claims will be exchanged for a fraction

q of the equity shares, while the current equityholders retain the remaining fraction of the equity shares, where q is endogenously determined. If the creditors do not accept this offer, the equityholders commit to declaring default to threaten the creditors. If this default event indeed occurs, creditors will take over the firm's asset and earn a fraction α of the first-best value of the asset through liquidation. We will later see that the debt-equity swap offer is designed to be always accepted in equilibrium.

The parameter q, that is, the fraction of equity shares that is exchanged for debt under renegotiation, is determined through the Nash bargaining between equityholders and creditors. The bargaining power of equityholders is β and that of creditors is $1 - \beta$. That is, a fraction β of the total surplus created through renegotiation will be acquired by equityholders and the remaining fraction of the total surplus will be acquired by creditors. In equilibrium, equityholders decide to initiate renegotiation when their firm's fundamental x_t falls to some threshold, say, x_S, which is endogenously determined.

In this model, an equilibrium is defined as the pair of x_S and x_R such that (i) each creditor optimally decides when to run by taking the withdrawal strategies of other creditors and the renegotiation strategy of equityholders as given and (ii) equityholders optimally decide when to initiate renegotiation by taking the withdrawal strategies of creditors. Here, note that the corner case of $x_S = x_R$, which means creditors never run before equityholders initiate renegotiation, can potentially arise. But we mainly pay attention to a more general case of $x_S < x_R$ throughout the paper because the corner case does not provide any interesting implications.

3 Model Solutions

In this section, we pin down the equilibrium of this model. We first consider the Nash bargaining game between equityholders and creditors under renegotiation. We then consider the optimal withdrawal strategy of each creditor and the optimal renegotiation strategy of equityholders.

Recall that equityholders initiate renegotiation when the firm's fundamental hits the threshold x_S. The total surplus created by the renegotiation is

$$V^{FB}(x_S) - \alpha V^{FB}(x_S) = \frac{(1-\alpha)x_S}{r-\mu}$$

because equityholders have committed to declaring default if creditors do not accept the renegotiation offer. Then, because equityholders have a bargaining power β, the fraction of equity shares that should be transferred to creditors through renegotiation must satisfy the following condition:

$$\frac{(1-q)x_S}{r-\mu} = \frac{\beta(1-\alpha)x_S}{r-\mu},$$

which means the amount of surplus acquired by equityholders is a fraction β of the total surplus. Here, we have used the fact that once the debt-equity swap is executed, the firm will be operated without any debt claims and thus, the equity value after renegotiation should be just equal to the first-best firm value. The above condition implies

$$q = 1 - \beta(1-\alpha). \tag{1}$$

This result certainly implies that the amount of surplus enjoyed by creditors is equal to a fraction $1-\beta$ of the total surplus.

We now consider each creditor's individual problem. To begin with, let $D(x_t)$ denote the present value of debt at time t. Then, note that each awakened creditor optimally chooses to run if and only if $D(x_t)$ is lower than the principal amount F. Hence, the individual problem of each creditor, who takes the run threshold of other creditors and the renegotiation threshold of equityholders as given, can be described as the following Hamilton-Jacobi-Bellman (HJB) equation:

$$rD(x) = c + \theta 1_{x<x_R}(L(x) - D(x)) + \lambda \max_{\text{run or stay}} \{F - D(x), 0\} + \mu x D_x(x) + \frac{\sigma^2}{2} x^2 D_{xx}(x) \tag{2}$$

subject to

$$D(x_S) = \frac{q x_S}{r-\mu}. \tag{3}$$

The left-hand side of (2) denotes the required return. The first term on the right-hand side is the coupon payment. The second term indicates the liquidity-driven default caused by the runs of other creditors. The third term means that each awakened creditor decides to run if and only if the current debt value is lower than the principal amount. The remaining terms indicate the expected changes in the debt value due to the time-varying fundamental. The boundary condition means that when the renegotiation is proposed, creditors always accept the offer and their debt contracts are swapped for a fraction q of the equity shares. In a symmetric equilibrium, the conjectured run threshold of other creditors, x_R, should be also individually optimal for each creditor. Hence, we must have

$$D(x_R) = F \tag{4}$$

in equilibrium.

Now let $E(x_t)$ denote the present value of equity at time t. Then we see that the equity value satisfies the following HJB equation:

$$rE(x) = x - c + \lambda 1_{x<x_R}(D(x) - F) + \theta 1_{x<x_R}(H(x) - E(x)) + \mu x E_x(x) + \frac{\sigma^2}{2} x^2 E_{xx}(x) \tag{5}$$

subject to

$$E(x_S) = \frac{(1-q)x_S}{r-\mu} \quad \text{and} \quad E_x(x_S) = \frac{1-q}{r-\mu}. \tag{6}$$

The left-hand side of (5) indicates the required return. The first term on the right-hand side is the amount of cash flows produced today. The second term represents the coupon payment. The third term means that once the firm successfully pays back the principal amount to all withdrawing creditors, the firm replaces the old debt claim with a new debt claim to fix the total size of debt. The fourth term indicates the liquidity-driven default event. The remaining terms denote the expected changes in the equity value due to the time-varying fundamental. The first boundary condition means that equityholders will maintain a fraction $1-q$ of the equity shares when the debt contracts are swapped for the equity shares. The second boundary condition is the so-called smooth-pasting condition, which should be understood as the condition for the optimal renegotiation timing.

We henceforth note that to pin down an equilibrium, we need to find $D(x)$, $E(x)$, x_S, and x_R that jointly satisfy the conditions from (2) to (6). In Appendix A.1, we solve for $D(x)$ and $E(x)$ in closed form for any given x_S and x_R. We pin down the equilibrium thresholds x_S and x_R numerically using a tractable system of linear equations. For the latter purpose, we define the firm value as $V(x) = E(x) + D(x)$.

4 Policy Implications

In this section, we discuss the model implications, particularly paying attention to the role of liquidity injection policies. To this aim, we first choose reasonable parameter values to present the model implications not only qualitatively but also quantitatively.

4.1 Parameter Values

We choose the parameter values as follows. We set the interest rate r to 3% because the average 10-year Treasury rate over the period from 2000 to 2019 was about 3.42%. We set the asset growth rate μ to 1% because, in the risk-neutral world, the asset growth rate is the risk-free rate minus the asset payout ratio, which is estimated to be 2% by Zhang et al. (2009). We set the asset volatility σ to 25% because the average asset volatility is about 22% according to Zhang et al. (2009). We normalize the principal amount of debt, F, to 100. We set the coupon payment c to 5 because the coupon rate that is widely used in the quantitative credit-risk literature ranges from 4% to 7%. We set the asset recovery rate α to 60% because the average asset recovery rate is about 55% or 60% according to Alderson and Betker (1995), Chen (2010), and Glover (2016). We set the parameter λ to 0.5, which means each creditor makes the withdrawal decision once per two years on average, because according to Bao et al. (2011), the average turnover rate of corporate bonds is about 0.42 based on the Trace Reporting and Compliance Engine (TRACE) data. We set the parameter θ which measures the fragility of emergency funding, to 0.2 because according to Schroth et al. (2014), the reasonable value for this parameter when firms have strong credit guarantees is about 0.14. We do not specifically choose the parameter value for the bargaining power of equityholders because we will examine the effects of liquidity injection policies by varying the values of this

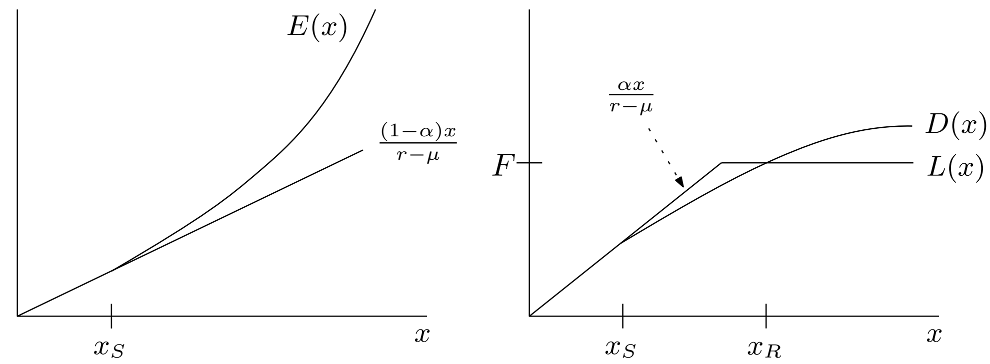

Figure 1: The left panel plots the equity value when the bargaining power of equityholders is equal to 1. The right panel plots the debt value in this case. **Source(s)**: By the authors.

parameter. The parameter values chosen above are summarized as follows:

$$r = 3\%,\ \mu = 1\%,\ \sigma = 25\%,\ F = 100,\ c = 5,\ \alpha = 60\%,\ \lambda = 0.5,\ \theta = 0.2. \tag{7}$$

Unless otherwise stated, these baseline parameter values are used in the numerical results presented below.

4.2 Effectiveness of Liquidity Injections

In this section, we present the key implications of our model. Specifically, using our model economy, we examine whether liquidity injections into borrowing firms can effectively mitigate runs and stabilize the debt market, especially when equityholders have a renegotiation option. We will see that the most intriguing result arises when creditors have no bargaining power. So, throughout this section, we focus on the case of $\beta = 1$. We will examine the other cases when equityholders have a different level of bargaining power in the next section.

When equityholders possess all the bargaining power, the debt value can be lower than the potential liquidation value of the asset. To see why, when β is 1, the fraction of equity shares that equityholders can retain through renegotiation, that is, $1 - q$, is equal to $1 - \alpha$, which can be seen from (1). Then, since equityholders optimally choose the renegotiation timing, the equity value should be always higher than $\frac{(1-\alpha)x}{r-\mu}$ as shown in the left panel of Figure 1. Then, since the firm value, that is, $V(x) = E(x) + D(x)$, is lower than the first-best value of the asset due to the liquidity-driven default event in our model, the debt value should be always lower than $\frac{\alpha x}{r-\mu}$ as depicted in the right panel of Figure 1. Note that the debt value can be strictly lower than the potential liquidation value because when the firm's fundamental is high enough, it is not optimal for the equityholders to renegotiate and therefore, the equity value lies strictly above $\frac{(1-\alpha)x}{r-\mu}$ in that case. Also, for clarification, the debt is not necessarily tangent to the line $\frac{\alpha x}{r-\mu}$ at the renegotiation threshold x_S, while the equity value is tangent to the line $\frac{(1-\alpha)x}{r-\mu}$ at that point due to the optimality condition.

The fact that the debt value can be lower than the liquidation value of the asset has important implications for a liquidity injection policy. When the debt value is lower than the potential liquidation value, an earlier default can be actually beneficial to creditors. More precisely, recall that $L(x)$ denotes the bankruptcy proceeds that creditors will earn when their firm fails to meet the early redemption requests due to a liquidity reason. Then, since the amount of these bankruptcy proceeds to creditors is equal to the liquidation value of the asset when the firm's fundamental is lower than the run threshold x_R, at least in this case, the debt value should be lower than the bankruptcy proceeds $L(x)$ as well, as shown in the right panel of Figure 1. But when the debt value is less than the bankruptcy proceeds to creditors, which particularly happens when the firm is facing runs, creditors will be better off if their firm defaults earlier. Put differently, if the government injects emergency liquidity into the debt market, such interventions may rather trigger more aggressive runs. More specifically, if a firm's default is deterred due to liquidity injections, then creditors will be worse off, especially when the potential liquidation value is higher than the current value of debt, and therefore the debt value will fall, causing more runs on the firm.

This observation implies that emergency capital injection programs, such as the Term Asset-Backed Securities Loan Facility, Commercial Paper Funding Facility, or the Secondary Market Corporate Credit Facility deployed by the Federal Reserve during the 2008 financial crisis and the 2020 COVID-19 crisis, may rather cause more frenzy runs from creditors and cause an earlier market collapse. In this regard, we argue that whether the government's bailout or liquidity injection programs can effectively improve market stability deserves a more thorough investigation.

To quantitatively measure these undesirable effects of liquidity injections, we examine how the parameter θ that measures the fragility of market liquidity affects debt markets, using

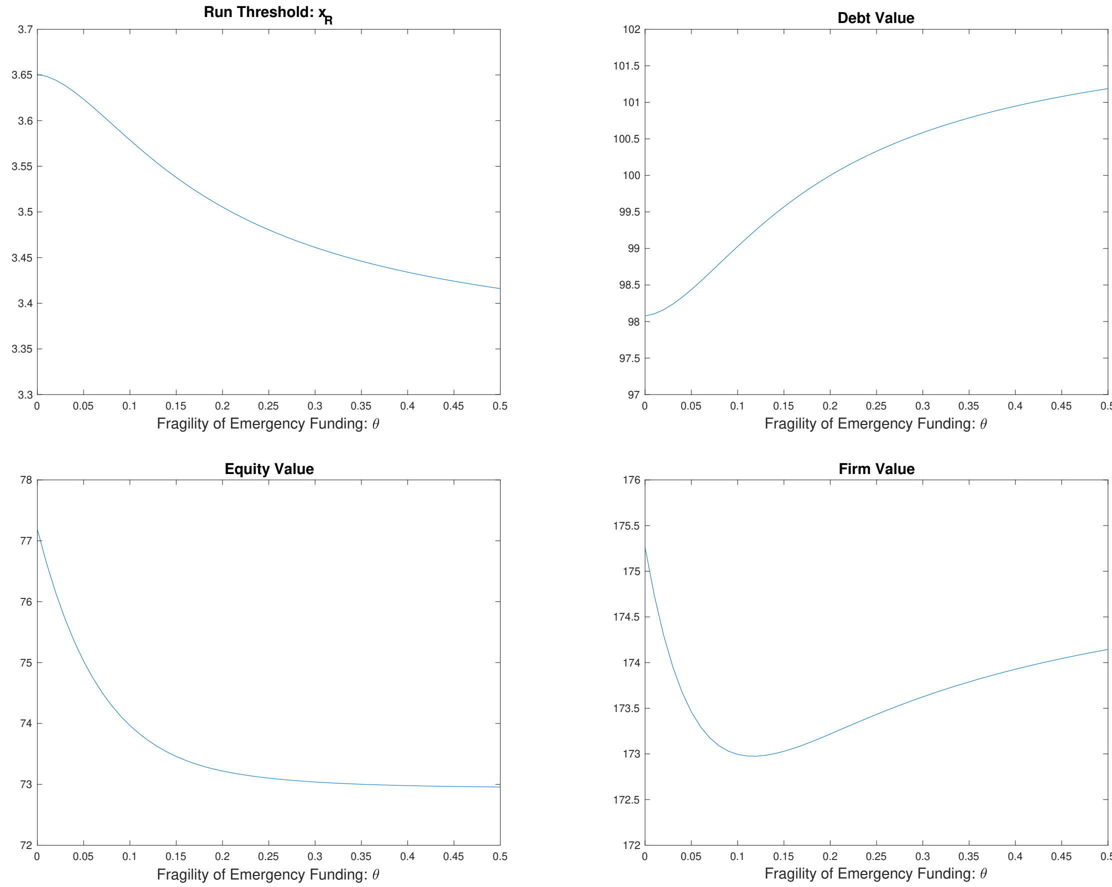

Figure 2: This figure plots the effect of the fragility of emergency funding when the bargaining power of equityholders is equal to 1. The top-left graph plots the effect on the run threshold x_R. The top-right graph plots the effect on the debt value. The bottom-left graph plots the effect on the equity value. The bottom-right graph plots the effect on the firm value. The firm's current fundamental x_0 is chosen to be 3.51 that corresponds to x_R under the baseline parameter values. **Source(s)**: By the authors.

the baseline parameter values summarized in (7). Figure 2 presents the numerical results of this comparative statics analysis. The top-left panel indeed shows that creditors decide to run more aggressively as the firm has access to stronger emergency funding. The top-right panel also shows that as the fragility of emergency funding decreases from 0.5 to 0, the debt value rather falls by about 3%. In particular, according to Schroth et al. (2014), the fragility parameter for strong credit guarantees is about 0.14 and that for weak credit guarantees is about 0.45. According to our numerical computations, as the fragility of emergency funding declines from 0.45 to 0.14, the debt value is lowered by about 2%.

Meanwhile, the bottom-left panel shows that the equity value increases as the firm faces a less liquidity problem. This result is intuitive because even if the improved liquidity may cause creditors to run more aggressively, when a firm becomes less likely to default due to the liquidity problem, at least the equityholders should be better off because early liquidation is indeed costly for equityholders. Nonetheless, the bottom-right panel shows that the overall effect of the improved liquidity on the firm value can be negative, which means that liquidity injections can indeed hurt the total welfare of the economy. In particular, as the fragility of emergency funding declines from 0.45 to 0.14, the firm value decreases by approximately 1%. Here, note that when θ is less than 0.07, the firm value increases as θ decreases. This result makes sence because the firm value is maximized when $\theta = 0$, in which case, any inefficient liquidity-driven default does not occur. So, one may say that if the government injects substantially large amounts of liquidity into the market, the total welfare will increase as desired. Nonetheless, in many cases, the government can provide only a limited amount of liquidity due to resource constraints. In such a realistic case, the total welfare may fall if the government injects some amount of emergency capital, as we have shown above.

The above result does not generally occur when equityholders do not have the renegotiation option. Specifically, in that case, unlike in Figure 1, the debt value is not necessarily pushed below the potential liquidation value of the asset, even when the firm's fundamental is close to the default boundary. As such, in this case, an earlier default will be indeed detrimental to creditors and therefore, the government's bailout programs seeking to strengthen the emergency funding will generally benefit creditors.

Figure 3 shows the effect of strengthening emergency funding when equityholders do not have the renegotiation option. As expected, when the fragility of emergency funding is lowered from 0.5 to 0, creditors run less aggressively and both the debt value and the equity value (and thus, the firm value as well) increase. This result shows that the presence of the renegotiation option plays a crucial role in generating the result regarding the negative effect of liquidity injections.

For clarification, solving the model without the renegotiation option is straightforward. That is, we can merely replace the boundary conditions for the equity value in (6) by $E(x_S) = E_x(x_S) = 0$. We do not present the closed-form solutions for this model for brevity.

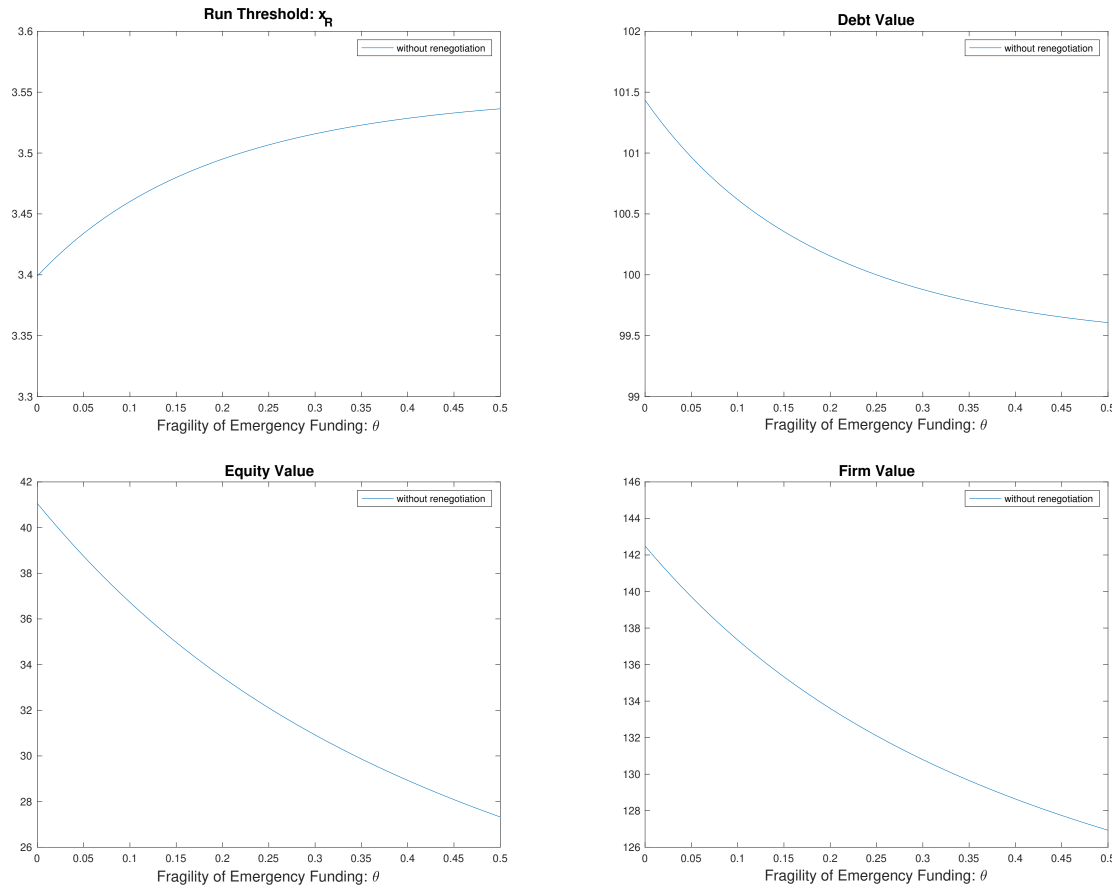

Figure 3: This figure plots the effect of the fragility of emergency funding when equityholders do not have the renegotiation option. The top-left graph plots the effect on the run threshold x_R. The top-right graph plots the effect on the debt value. The bottom-left graph plots the effect on the equity value. The bottom-right graph plots the effect on the firm value. The firm's current fundamental x_0 is chosen to be 3.51 which corresponds to x_R under the baseline parameter values in the model without the renegotiation option. **Source(s)**: By the authors.

4.3 Effect of Bargaining Power

In the previous section, we have assumed that the bargaining power of equityholders is equal to 1 to highlight the main result of our model. We now show that when equityholders do not have the full bargaining power, the undesirable negative effect of liquidity injections is less likely to occur. Specifically, when the bargaining power of equityholders is less than 1, the fraction of equity shares that creditors will receive through renegotiation will be higher than α, as can be seen from (1). Hence, in this case, the debt value is not necessarily lower than the potential liquidation value of the asset, similar to the case without the renegotiation option. Therefore, the provisions of emergency funds by the government can alleviate the

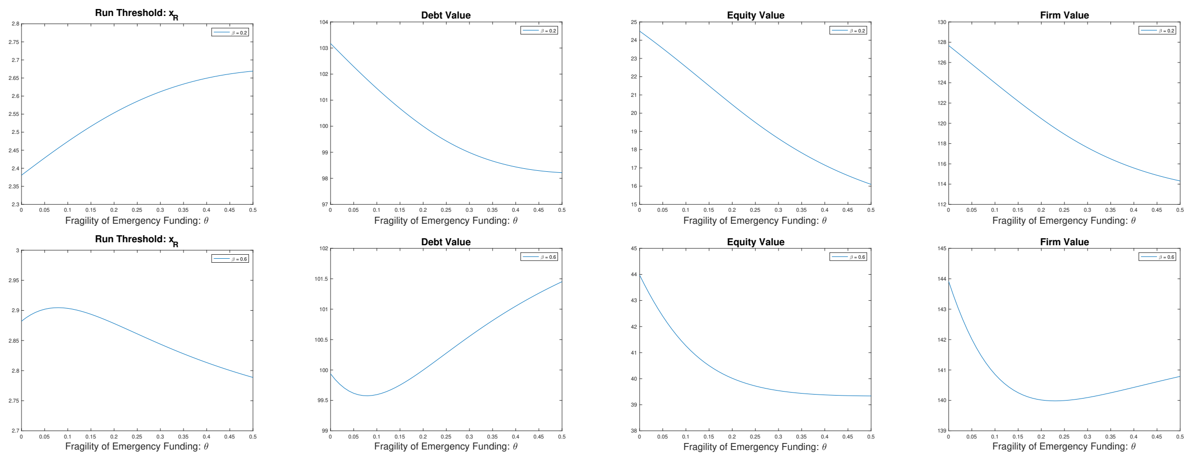

Figure 4: The upper four graphs plot the effect of the fragility of emergency funding when the bargaining power of equityholders is equal to 0.2. The bottom four graphs plot the effect of the fragility of emergency funding when the bargaining power of equityholders is equal to 0.6. In both cases, the first graph plots the effect on the run threshold x_R. The second graph plots the effect on the debt value. The third graph plots the effect on the equity value. The fourth graph plots the effect on the firm value. In all the graphs, the firm's current fundamental x_0 is chosen to be 3.51 which corresponds to x_R under the baseline parameter values. **Source(s)**: By the authors.

runs from creditors and boost the debt value, as desired.

The upper fourth graphs in Figure 4 show the effect of liquidity injections on debt markets when equityholders have a bargaining power of 0.2. In this case, when the fragility of emergency funding declines from 0.5 to 0, creditors delay their redemption request and therefore, the debt value, equity value, and firm value unanimously increase. However, when equityholders have an intermediate level of bargaining power, a mixed result arises. Specifically, the lower four graphs in Figure 4 show the effect of injections of emergency funding when the bargaining power of equityholders is 0.6. In this case, as the figure shows, when a firm has access to stronger emergency funding, liquidity injections tend to benefit creditors. But when a firm has weaker emergency funding, the same policy tends to hurt creditors, as in the case where equityholders have the full bargaining power.

5 Renegotiation via Strategic Debt Service

In the main model, we have assumed that equityholders renegotiate the debt contract terms with creditors through a debt-equity swap. In this section, we consider an alternative renegotiation scheme known as strategic debt service examined in Mella-Barral and Perraudin (1997).

Under this alternative renegotiation scheme, equityholders can adjust the debt contract terms at each point in time instead of offering a one-time debt-equity swap. For simplicity, as in Anderson and Sundaresan (1996), we assume that equityholders can adjust only the coupon payment, not the principal amount or the maturity date. As such, let $s(x_t)$ denote the adjusted debt service flow at time t. Then, we can reasonably postulate that equityholders adjust the coupon payment if and only if their firm's fundamental is below some threshold, denoted by x_S. Also, for simplicity, as in Mella-Barral and Perraudin (1997), we assume that equityholders have the full bargaining power.[2]

Besides the renegotiation scheme, the other features of the model are the same as those described in the main model. That is, each creditor makes the withdrawal decision according to an idiosyncratic Poisson shock that arrives with an intensity λ. Also, the firm fails to meet the early redemption requests in the Poisson manner with an intensity θ.

In this setup, because creditors have no bargaining power, equityholders optimally choose $s(x)$ in a way that the debt value is pushed down to the potential liquidation value of the asset, especially when the firm's fundamental is below x_S. Equityholders cannot push down the debt value further because creditors would then reject the offer. Also, since equityholders cannot adjust the principal amount or the debt maturity, when the firm's fundamental is substantially low, the net cash flow to equity will be negative even if the equityholders have the renegotiation option. Hence, we can also postulate that equityholders eventually decide to default when their firm's fundamental hits a default boundary x_D, which is endogenously determined.

Given this observation, the debt value satisfies the following HJB equation:

$$rD(x) = s(x)1_{x<x_S} + c1_{x\geq x_S} + \theta 1_{x<x_R}(L(x) - D(x)) + \lambda \max\{F - D(x), 0\} + \mathcal{A}D(x),$$

[2]In the literature, Fan and Sundaresan (2000) also consider the setting where equityholders have an intermediate level of bargaining power in the context of strategic debt service. But incoporating the intermediate level of bargaining power when creditors also have the withdrawal option seems to be technically challenging.

subject to

$$D(x_D) = \frac{\alpha x_D}{r-\mu}.$$

We can use this HJB equation to precisely determine the strategic debt service flow, $s(x)$, when the firm's fundamental is below x_S. Specifically, recall that the debt value is pushed down to the potential liquidation value of the asset due to renegotiation. Hence, when x is lower than x_S, the above HJB equation can be rewritten as

$$\frac{r\alpha x}{r-\mu} = s(x) + \lambda\left(F - \frac{\alpha x}{r-\mu}\right) + \frac{\mu\alpha x}{r-\mu},$$

which implies

$$s(x) = \alpha x - \lambda\left(F - \frac{\alpha x}{r-\mu}\right), \quad \forall x \in (x_D, x_S).$$

We now note that the equity value satisfies

$$rE(x) = x - s(x)1_{x<x_S} - c1_{x\geq x_S} + \theta 1_{x<x_R}(H(x) - E(x)) + \lambda 1_{x<x_R}(D(x) - F) + \mathcal{A}E(x),$$

subject to

$$E(x_D) = E_x(x_D) = 0.$$

In equilibrium, since the conjectured run threshold of other creditors must be the same as the run threshold of any individual creditor, the equilibrium threshold x_R must satisfy

$$D(x_R) = F.$$

Although we can solve this model in closed form, we omit to do this task for brevity.

The main result of our paper still holds under this alternative renegotiation scheme. The underlying intuition is clear. Since the debt value is again reduced below the potential liquidation value of the firm, injecting liquidity into debt markets to help distressed firms survive longer may not necessarily benefit creditors.

6 Conclusion

In this paper, we developed a dynamic debt-run model with a renegotiation option and show that liquidity injections into debt markets may rather cause creditors to run more aggressively. This undesirable outcome is more likely to occur when creditors have a lower bargaining power. Specifically, in that case, the debt value is generally pushed below the potential liquidation value of the firm's asset and therefore, injecting emergency funds into liquidity-constrained firms to defer a default can be rather harmful to creditors. In this regard, conducting empirical studies to investigate whether liquidity backstop programs indeed caused different effects on firms, depending on the bargaining power of equityholders of those firms will be meaningful. We can potentially measure the bargaining power of equityholders using the data about how often renegotiations were initiated or the actual amount of bankruptcy proceeds accrued to equityholders in the event of default.

A Appendix

A.1 Closed-form Solutions

In this section, we solve the model in closed form. To this aim, let x_L denote the point that satisfies

$$\frac{\alpha x_L}{r-\mu} = F.$$

That is, when the firm defaults due to a liquidity reason, the bankruptcy proceeds to creditors will be capped by the principal amount when the firm's fundamental is higher than x_L. Then we consider two cases: (i) $x_S < x_L < x_R$ and (ii) $x_S < x_R < x_L$. We do not consider other cases where x_S is larger than x_L or is equal to x_R because those trivial cases do not yield any interesting results.

Case (i): When $x_S < x_L < x_R$, the value of debt is written as

$$D(x) = \begin{cases} C_1 + C_2 x + A_1 x^{\eta_1} + A_2 x^{\eta_2}, & \text{if } x_S \le x < x_L \\ C_3 + A_3 x^{\eta_1} + A_4 x^{\eta_2}, & \text{if } x_L \le x < x_R \\ C_4 + A_5 x^{\eta_3}, & \text{if } x_R \le x, \end{cases}$$

where

$$C_1 = \frac{c + \lambda F}{r + \lambda + \theta}, \quad C_2 = \frac{\theta\alpha}{(r - \mu)(r + \lambda + \theta - \mu)}, \quad C_3 = \frac{c + \lambda F + \theta F}{r + \lambda + \theta}, \quad C_4 = \frac{c}{r},$$

$$\eta_1, \eta_2 = \frac{-\mu + \frac{\sigma^2}{2} \pm \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2\sigma^2(r + \lambda + \theta)}}{\sigma^2},$$

$$\eta_3 = \frac{-\mu + \frac{\sigma^2}{2} - \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2\sigma^2 r}}{\sigma^2}.$$

The coefficients A_1, ..., A_5 are determined from the following boundary conditions:

$$D(x_S) = qV^{FB}(x_S), \ \lim_{x \uparrow x_L} D(x) = \lim_{x \downarrow x_L} D(x), \ \lim_{x \uparrow x_L} D_x(x) = \lim_{x \downarrow x_L} D_x(x),$$
$$\lim_{x \uparrow x_R} D(x) = \lim_{x \downarrow x_R} D(x), \ \lim_{x \uparrow x_R} D_x(x) = \lim_{x \downarrow x_R} D_x(x).$$

As these boundary conditions lead to a system of linear equations, we can solve for the coefficients A_1, ..., A_5 explicitly.

Next, the equity value is given by

$$E(x) = \begin{cases} \frac{-c+\lambda(C_1-F)}{r+\theta} + \frac{(1+\lambda C_2)x}{r+\theta-\mu} + \sum_{i=1}^{2} \frac{\lambda A_i x^{\eta_i}}{g(\eta_i)} + B_1 x^{\xi_1} + B_2 x^{\xi_2}, & \forall x \in [x_S, x_L) \\ \frac{-c+\lambda(C_3-F)-\theta F}{r+\theta} + \frac{\left(1+\frac{\theta\alpha}{r-\mu}\right)x}{r+\theta-\mu} + \sum_{i=1}^{2} \frac{\lambda A_{i+2} x^{\eta_i}}{g(\eta_i)} + B_3 x^{\xi_1} + B_4 x^{\xi_2}, & \forall x \in [x_L, x_R) \\ -\frac{c}{r} + \frac{x}{r-\mu} + B_5 x^{\eta_3}, & \forall x \in [x_R, \infty), \end{cases} \tag{8}$$

where

$$g(\eta) = r + \theta - \mu\eta - \sigma^2\eta(\eta - 1)/2,$$

$$\xi_1, \xi_2 = \frac{-\mu + \frac{\sigma^2}{2} \pm \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2\sigma^2(r + \theta)}}{\sigma^2}.$$

The coefficients B_1, ..., B_5 are determined from the following boundary conditions:

$$E(x_S) = (1 - q)V^{FB}(x_S), \ \lim_{x \uparrow x_L} E(x) = \lim_{x \downarrow x_L} E(x), \ \lim_{x \uparrow x_L} E_x(x) = \lim_{x \downarrow x_L} E_x(x),$$
$$\lim_{x \uparrow x_R} E(x) = \lim_{x \downarrow x_R} E(x), \ \lim_{x \uparrow x_R} E_x(x) = \lim_{x \downarrow x_R} E_x(x).$$

As these boundary conditions lead to a system of linear equations, we can solve for the coefficients B_1, ..., B_5 explicitly.

The equilibrium thresholds, x_S and x_R, are then pinned down from the following conditions:

$$D(x_R) = F \quad \text{and} \quad E_x(x_S) = \frac{(1-q)}{r-\mu}. \tag{9}$$

We compute the pair of the equilibrium thresholds numerically by solving these two conditions.

Case (ii): When $x_S < x_R < x_L$, the solutions of the model have simpler form. Specifically, the value of debt is written as

$$D(x) = \begin{cases} C_1 + C_2 x + A_1 x^{\eta_1} + A_2 x^{\eta_2}, & \text{if } x_S \leq x < x_R \\ C_3 + A_3 x^{\eta_3}, & \text{if } x_R \leq x, \end{cases}$$

where

$$C_1 = \frac{c+\lambda F}{r+\lambda+\theta}, \quad C_2 = \frac{\theta\alpha}{(r-\mu)(r+\lambda+\theta-\mu)}, \quad C_3 = \frac{c}{r},$$

The coefficients A_1, A_2, and A_3 are determined from

$$D(x_S) = qV^{FB}(x_S), \ \lim_{x\uparrow x_R} D(x) = \lim_{x\downarrow x_R} D(x), \ \lim_{x\uparrow x_R} D_x(x) = \lim_{x\downarrow x_R} D_x(x).$$

As these boundary conditions lead to a system of linear equations, we can solve for the coefficients A_1, A_2, and A_3 explicitly.

Next, the equity value is given by

$$E(x) = \begin{cases} \frac{-c+\lambda(C_1-F)}{r+\theta} + \frac{(1+\lambda C_2)x}{r+\theta-\mu} + \sum_{i=1}^{2} \frac{\lambda A_i x^{\eta_i}}{g(\eta_i)} + B_1 x^{\xi_1} + B_2 x^{\xi_2}, & \forall x \in [x_S, x_R) \\ -\frac{c}{r} + \frac{x}{r-\mu} + B_3 x^{\eta_3}, & \forall x \in [x_R, \infty). \end{cases}$$

The coefficients B_1, B_2, and B_3 are determined from the following boundary conditions:

$$E(x_S) = qV(x_S), \ \lim_{x\uparrow x_R} E(x) = \lim_{x\downarrow x_R} E(x), \ \lim_{x\uparrow x_R} E_x(x) = \lim_{x\downarrow x_R} E_x(x).$$

As these boundary conditions lead to a system of linear equations, we can solve for the coefficients B_1, B_2, and B_3 explicitly.

As in the previous case, the equilibrium thresholds, x_S and x_R, are pinned down from the two conditions in (9). We compute the pair of the equilibrium thresholds numerically by using these two conditions.

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